Test 6
Continuous Random Variables

This assessment contributes 7% towards the final year mark.
Time Allowed : 40 minutes
This is a Calculator-Assumed Task.
No notes of ANY nature are permitted for this assessment.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name : Solutions

Score : (out of 35)

Do not turn over this page until you are instructed to do so.

1. **(5 marks)**

cumulative

The lifetimes (t hours) of a consignment of electric light bulbs are displayed below.

t	$t \le 50$	$t \le 100$	$t \le 150$	$t \le 200$	$t \le 250$	$t \le 300$	$t \le 350$	$t \le 400$
Number of bulbs	8	20	60	180	250	300	380	410
	8	12	40	120	70	50	80	30

Define the random variable T: Lifetime of light bulbs.

Estimate

a) The mean and variance of T. (2 marks)

Syllabus: use RF and histograms to find probability

t	no of bulbs
0 – 50	8
50 – 100	12
100 – 150	40
150 – 200	120
200 – 250	70
250 – 300	50
300 – 350	80
350 – 400	30
	410

Now CAS

Relative Frequency

0.1951219512

0.07317073171

CAS | stats

25	8
75	12
125	40
175	120
225	70
275	50
325	80
375	30

calc 1var Freq(list 2)

Mean

228.90 hours

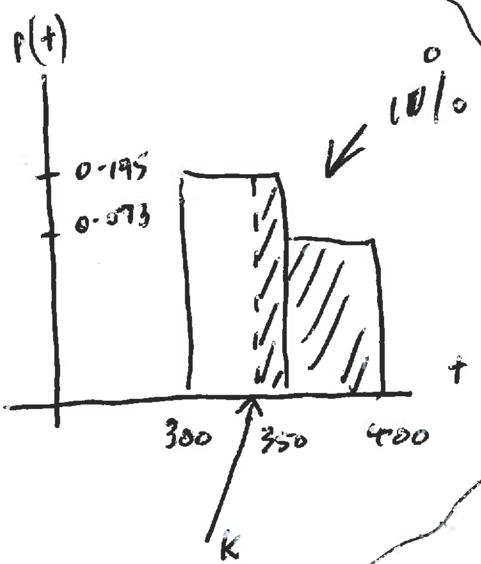
Variance

$(83.866271)^2$

$= 7033.55$

b) The lifetime exceeded by 10% of the light bulbs. (3 marks)

$P(T \geq k) = 0.9$



Relative Frequency

$0.1 - 0.07317073...$

$= 0.026829268 29$

$\times 410$

$= 11$

$k = 350 - \left(\frac{11}{80} \times 50\right)$

$= 343.125$

$\therefore$ Lifetime 343.125 hours

answer

2. **(7 marks)**

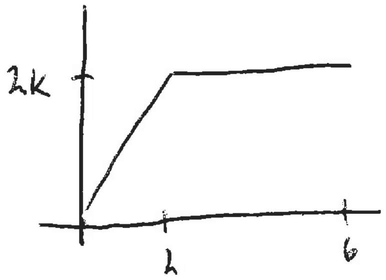
A continuous random variable, X, has a probability density function given

by $$f(x)=\begin{cases} kx & 0\le x\le 2 \\ 2k & 2<x\le 6 \\ 0 & otherwise \end{cases}$$

or $\int_0^2 kx\,dx + \int_2^6 2k\,dx = 1$

concept of area

a) Determine the value of k. (3 marks)



Area $= 1$

$A = \frac{1}{2}(2)(2k) + 4 \times (2k)$ find area

$= 2k + 8k$

$10k = 1$ $\therefore k = 0.1$ answer

b) Find

i) $P(X \le 4)$ $= 2k + 4k$ (1 mark)

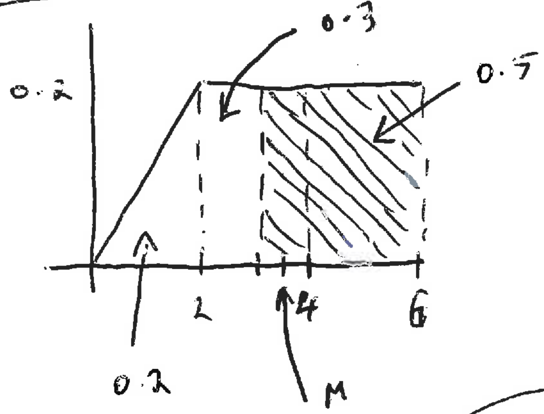
$= 0.6$ answer

or $\int_0^2 \frac{1}{10}x\,dx + \int_2^4 \frac{2}{10}\,dx$

or $\int_M^6 \frac{1}{5}\,dx = 0.5$

ii) ~~M, the~~ median of the distribution. (3 marks)

or $(M-2) \times 0.2 = 0.3$



$P(X \le M) = 0.5$ concept of median

$(6-M) \times 0.2 = 0.5$ equation

$6 - M = 2.5$

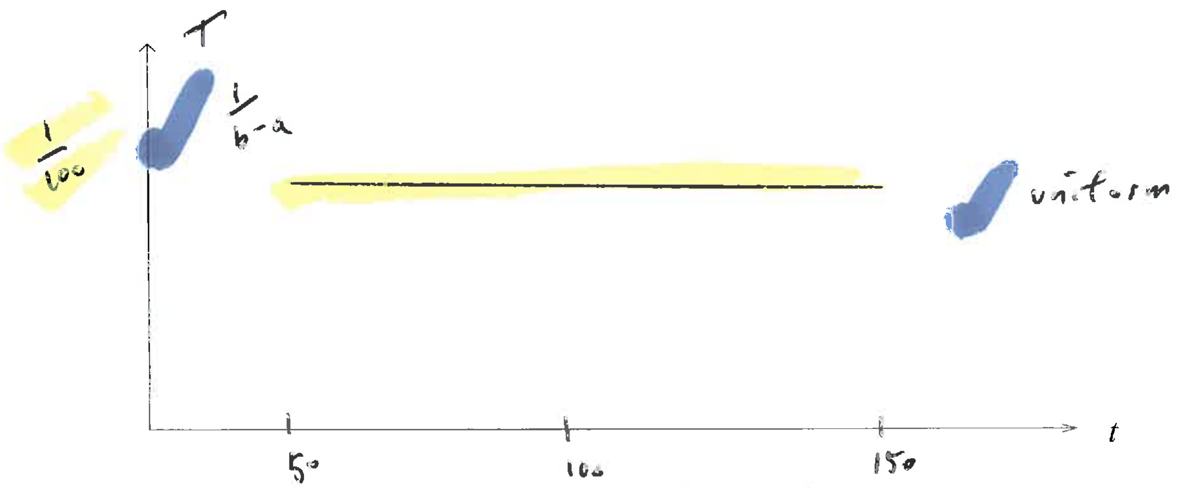
$M = 3.5$ answer

or $\int_2^M \frac{1}{5}\,dx = 0.3$

3. (8 marks)

~~The serving~~ time, T seconds, for a customer at an automatic banking machine is a uniformly distributed random variable, with lower and upper limits 50 and 150. The mean ~~serving~~ time is 100 seconds and the standard deviation is 28.9 seconds. The serving times for different customers are independent.

a) Sketch the graph of the distribution function for T. (2 marks)



b) Evaluate $P(T = 120)$. $= 0$ answer (1 mark)

c) Evaluate $P(T \geq 120)$. $30 \times \frac{1}{100} = 0.3$ answer (1 mark)

or $\int_{120}^{150} 0.01 \, dt$

d) What is the probability that exactly 3 of the next 5 customers will require at least 2 minutes to be served? (2 marks)

$Y \sim B(5, 0.3)$

Y is no of customers requiring at least 2 min out of 5

parameters

$P(Y = 3) = 0.1323$

answer

Bin CDF(3,3,5, 0.3)

e) Find the cumulative distribution function for T. (2 marks)

$$P(T \leq x) = \int_{50}^{x} \frac{1}{100} \, dt$$

integral

$$= \left[\frac{t}{100} \right]_{50}^{x}$$

$$= \frac{x}{100} - \frac{50}{100} = \frac{x}{100} - \frac{1}{2}$$

answer

4. **(4 marks)**

The queuing time, X minutes, of a traveller at the ticket office of a large railway station has a probability density function f defined by

$$f(x)=\begin{cases}0.0004x(100-x^2) & 0\le x\le 10\\ 0 & otherwise\end{cases}$$

a) Find the mean of the distribution. (2 marks)

$$\mu = \int_0^{10} x \cdot 0.0004x(100-x^2)\,dx$$ concept

$$= 5.\dot{3} \text{ minutes}$$ answer

$\frac{16}{3}$

b) Find the standard deviation of the distribution, correct to 2 decimal places. (2 marks)

$$\text{Variance} = \int_0^{10} 0.0004x(100-x^2)\left(x-\frac{16}{3}\right)^2 dx$$ concept

$$= 4.\dot{8}$$

$$SD = 2.21 \text{ min } (2dp)$$ answer

or

$$\int_0^{10} x^2 \times 0.0004x(100-x^2)\,dx - 5.\dot{3}^2$$

5. (4 marks)

A continuous random variable X has a mean of 55 and a standard deviation of 5. With a change of scale and origin the random variable Y= aX + b. Find a and b if the mean and standard deviation of Y are 173 and 15 respectively.

Mean

$173 = a(55) + b$ ✓ Mean

$173 = 3(55) + b$

$b = 8$

SD

$15 = |a|(5)$ ✓ SD

$|a| = 3$

$a = \pm 3$

$\therefore$ $a = 3$ and $b = 8$ ✓

Or $a = -3$ and $b = 338$ ✓

6 **(7 marks)**

The Longlife Tyre Company produces a radial tyre which has a mean lifetime of 50 000 km and a standard deviation of 5 000 km. The lifetime of a tyre is normally distributed.

a) Find the probability that any one tyre will last longer than 40 000 km. (1 mark)

$X \sim N(50000, 5000^2)$

$P(X > 40000) = 0.9772499 \approx 0.9772$ (4dp)

answer

b) Find the probability that all four tyres bought by a particular customer will last more than 40 000 km. (2 marks)

$P(\text{all 4 tyres} > 40000) = 0.9772^4$ concept

$= 0.9121$ (4dp) answer

or $Y \sim Bin(4, 0.9772499)$

$P(Y = 4) =$ BinCDF(4, 4, 4, 0.9772....)

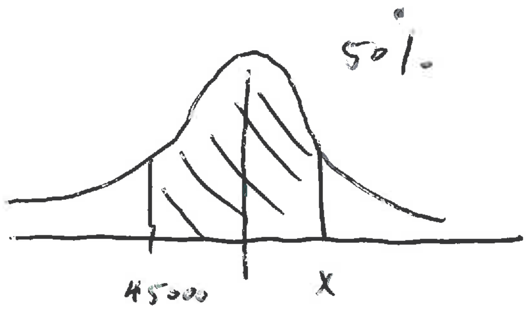
c) Find the probability that a tyre lasts more than 60 000 km given that it has already lasted 40 000 km. (2 marks)

$P(X > 60000 / X > 40000) = \dfrac{P(X > 60000)}{P(X > 40000)}$ numerator

$= \dfrac{0.0227501}{0.9772499} = 0.02328 \approx 0.0233$ answer

Don't round early

d) Approximately fifty percent of the tyres last between 45,000 km and x km. Find the value of x accurate to the nearest one hundred. (2 marks)



$P(X < 45000) = 0.1586553$ calculate

add 0.5

$P(X < x) = 0.6586553$

$x = 52043.48$

≈ 52000 tyres (nearest hundred) answer

or

solve(NormCDF(45000, x, 5000, 50000) = 0.5